Segmentation of Diffusion Tensor Images

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Brain Analysis
- Structures (corpus callosum, arcuate fasciculus, corona radiata) of the white matter
- Fiber bundles

Diagnose and locate diseases
- Strokes
- Tumors
- Alzheimer’s disease
- Schizophrenia
Examples

Figure: Nerve tracts

Figure: Corpus callosum
1 Motivation
2 Introduction
3 Segmentation of diffusion tensor images
4 Region-based active contour for DTI segmentation
   • Segmentation Models
   • Algorithm
5 Results
Specific magnetic resonance imaging modality
Non-invasive Method
Tries to estimate the diffusion of water molecules
Isotropic and anisotropic diffusion
Acquisition and estimation

- Acquisition with an MRI scanner
- At least 6 diffusion-weighted (and one unweighted image) images in non-collinear directions
- Diffusion gradient changed with magnetic field variations in the MRI magnet
- For each voxel, the diffusion tensor can be estimated with linear or variational estimation
Diffusion Tensor

\[
D = \begin{pmatrix}
D_{xx} & D_{xy} & D_{xz} \\
D_{yx} & D_{yy} & D_{yz} \\
D_{zx} & D_{zy} & D_{zz}
\end{pmatrix}
\]

- 3x3 symmetric matrix
- Positive-definite matrix
- Fiber direction: tensor’s main eigenvector
Overview

- K-means [7]
- Boundary-based active contours [8, 4]
- Region-based active contour [9]
K-means algorithm

1. Choose number of clusters and their centers
2. Assign a cluster to each data point due to a distance measure
3. Recompute cluster centers
4. Stop if no data point is assigned to a different cluster as the step before
K-means for DTI segmentation

Clustering measure between a voxel $j$ and a cluster $k$

$$E_{jk} = \|x_j - x_k\|_{Wk} + \gamma \|D_j - D_k\|_F$$

with the Mahalanobis voxel distance

$$\|x\|_{Wk} = \sqrt{x^T W^{-1} x}$$

and Frobenius distance between two diffusion tensors

$$\|D_1 - D_2\|_F = \sqrt{\sum_{ij} (D_{1,ij} - D_{2,ij})^2}$$
Boundary-based active contours for DTI segmentation (1/2)

Geometric active contours:

\[
\frac{\partial \phi}{\partial t} = -F \cdot \nabla \phi
\]

with zero level of function \( \phi \) is the evolving curve \( C \) and

\[
F = F_{\text{data}} + \beta F_{\text{curv}}
\]

is the speed of the evolving curve.
Geodesic active contours:

\[
\frac{\partial \phi}{\partial t} = g(\cdot) |\nabla \phi| \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|} + \nabla g(\cdot) \cdot \nabla \phi
\]

where \( g(\cdot) \) is a stopping function.

Gradient magnitude:

\[
\text{gradMag}(D_\sigma) ::= \sqrt{\sum_{ij} |\nabla D_{\sigma,ij}|^2}
\]
Region-based active contour based DTI segmentation

- use Region-based active contour models
- incorporate an information theoretic tensor dissimilarity measure based on Kullback-Leibler divergence
- are robust to noise and insensitive to initialization

There are 2 approaches which are based on

1. Geometric active regions developed by Lenglet et.al. [2] and Rousson et.al [5]
Mumford-Shah Segmentation Model

Minimization of following variational principle based on the Mumford-Shah functional:

\[ E(T, C) = \int_{\Omega} d^2(T(x), T_0(x)) dx + \alpha \int_{\Omega/C} p(T)(x) dx + \beta |C| \]

where
- \( C \) \ldots boundary of the unknown segmentation
- \( \Omega \in \mathbb{R}^2 \) \ldots image domain
- \( T_0 \) \ldots given noisy DTI
- \( T \) \ldots piecewise smooth approximation of \( T_0 \)
- \( |C| \) \ldots arc length of curve \( C \)
- \( \alpha, \beta \) \ldots control parameter
- \( d(., .) \) \ldots Diffusion tensor distance measure
Modification of the active contour model without edges by Chan and Vese [1]

Efficient segmentation of tensor fields with two constant regions

Functional:

\[ E(C, T_1, T_2) = \int_R d^2(T(x), T_1)dx + \int_{R^C} d^2(T(x), T_2)dx + \beta |C| \]

- \( R \) ... region enclosed by \( C \)
- \( R^C \) ... region outside \( C \)
- \( T_1, T_2 \) ... mean values of DTI in region \( R, R^C \)
Information theoretic diffusion tensor distance

- Kullback-Leibler divergence for two densities $p$ and $q$ is defined by
  \[
  KL(p\|q) = \int p(x) \log \frac{p(x)}{q(x)} \, dx
  \]

- J-divergence (symmetrized Kullback-Leibler) is defined by
  \[
  J(p, q) = \frac{1}{2} [KL(p\|q) + KL(q\|p)]
  \]

- Information theoretic diffusion tensor distance is defined by
  \[
  d(T_1, T_2) = \sqrt{J(p(r|t, T_1), p(r|t, T_2))}
  \]

- In case of Gaussian distributions it is given by
  \[
  d(T_1, T_2) = \frac{1}{2} \sqrt{\text{tr}(T_1^{-1} T_2 + T_2^{-1} T_1) - 2n}
  \]
  where $n$ is size of $T_1$ and $T_2$. 
Algorithm 1  Two Stage Piecewise Constant Segmentation of DTIs [6]

1: Set initial curve $C_0$ and compute its signed distance function $\phi_0$.
2: Compute T1 and T2 according to Theorem 1
3: Update signed distance function $\phi$.
4: Reinitialize $\phi$ using the updated zero level set.
5: Stop if the solution is achieved, else go to step 2.
Theorem 1: The mean value of a tensor field defined as

$$\bar{M}(T, R) = \min_{M \in SPD(n)} \int_{\mathbb{R}} d^2 [M, T(x)] \, dx$$

is given by

$$\bar{M} = \sqrt{B^{-1}} \left[ \sqrt{\sqrt{B}A\sqrt{B}} \right] \sqrt{B^{-1}}$$

where $A = \int_{\mathbb{R}} T(x) dx$ and $B = \int_{\mathbb{R}} T^{-1}(x) dx$ and $SPD(n)$ denotes the set of symmetric positive definite matrices of size $n$. 
Matrix diagonalization (1/2)

Matrix diagonalization to compute

\[
\tilde{M} = \sqrt{B^{-1}} \left[ \sqrt{\sqrt{BA\sqrt{B}}} \right] \sqrt{B^{-1}}
\]

- A real symmetric matrix A can be diagonalized

\[
A = ODO^T
\]

where O is an orthogonal matrix and D is a diagonal matrix: \( D = \{d_{11}, d_{22}, \ldots, d_{nn}\} \)

- Furthermore

\[
A^\alpha = OD^\alpha O^T
\]

where \( D^\alpha = \{d_{11}^\alpha, d_{22}^\alpha, \ldots, d_{nn}^\alpha\} \)
Algorithm 2 Computation of $\sqrt{\sqrt{BA}\sqrt{B}}$ [6]

1: Diagonalize $B = O_B D_B O_B^T$

2: Compute $\sqrt{B} = O_B \sqrt{D_B} O_B^T$

3: Compute $Q = \sqrt{BA} \sqrt{B} = O_B \sqrt{D_B} O_B^T A O_B \sqrt{D_B} O_B^T$

4: Diagonalize $Q = O_Q D_Q O_Q^T$

5: Compute $\sqrt{Q} = O_Q \sqrt{D_Q} O_Q^T$
Computation of $\phi$

- Curve evaluation formulated as Level Set Framework
- Signed distance function $\phi$ of $C$

$$\frac{\partial \phi}{\partial t} = \left[ \beta \nabla \frac{\nabla \phi}{|\nabla \phi|} - d^2(T, T_1) + d^2(T, T_2) \right] |\nabla \phi|$$
Segmentation using Piecewise Smooth Model

- Variation: Piecewise Smooth Model
- Smoothing stage: curve is fixed, smoothing inside and outside the curve

\[ E_C(T) = \int_{\Omega} d^2(T(x), T_0(x))dx + \alpha \int_{\Omega/C} p(T)(x)dx \]

- Curve evolution stage: the inside and outside of the smoothed tensor field are fixed, the curve is moved

\[ E_C(T) = \int_{R} d^2(T_R(x), T_0(x))dx + \int_{R^C} d^2(T_{R^C}(x), T_0(x))dx + \alpha \int_{R} p(T_R)(x)dx + \alpha \int_{R^C} p(T_{R^C})(x)dx + \beta |C| \]
Segmentation of a synthetic tensor field where two regions differs only in the orientations [6]:
Results

Segmentation of a synthetic noisy tensor field [6]:

(a)  
(b)  

(c)  
(d)
Results

A slice of the DTI of a normal rat spinal cord viewed using ellipsoids [6]:
Segmentation of the slice of DTI [6]:

(a)  
(b)
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Thanks for your attention!

Any questions?